1. Introduction
A major objective of plant and animal breeding programmes is to improve the genetic potential of plants and animals. The breeding experiments comprise of two types of designs namely, mating designs and environmental designs. Mating design is a procedure of producing the progenies. This, however, is only one component of the actual experiment since the offspring of these crosses have to be reared subjecting these progenies to the environmental conditions in a systematic manner by using an environmental design.

Diallel cross plans are one of the commonly used mating designs. With limited facilities available for testing, a diallel cross may only be possible for a relatively small number of inbred lines. However, if only a small number of lines are included, the estimate of the variance of the general combining abilities (gca) in the whole population of potentially available inbred lines is subject to a large sampling error and also, many potentially high yielding inbred lines may be left out completely untested. It is, therefore, necessary to have a large number of inbred lines but raise only a sample of all possible crosses among them. Such a diallel cross is known as partial diallel cross (PDC). Gilbert (1958), Hinkelmann and Stern (1960), Kempthorne and Curnow (1961) have discussed the advantages of performing only a sample of all possible crosses among a large number of parents rather than making all possible crosses among a smaller number of parents.

Through PDC, a plant breeder is not only able to estimate the gca of a large number of parental lines but can also make selection among crosses from a wide range of parents. GCA of each line may be estimated with relatively low precision but larger genetic gains may result from the more intense selection that can be applied to them. For enabling the plant breeders to make use of PDC in their experimentation, it is necessary to develop methods of their construction and analysis. It is equally important to indicate which of the several possible designs for PDC, in a given situation, is the most efficient in the sense that it gives the least average variance of the difference between gca effects of a pair of lines.

There are many different ways of approaching the problem of sampling a diallel cross. The first approach involves developing a PDC based on circulant structure (Kempthorne and Curnow, 1961). In this method, the line i is crossed with lines (k+i), (k+i+1),...(k+i+s-1) where \( k = (v+1-s)/2 \) and s is the number of times a line is involved in crosses. All the numbers above v are to be reduced to modulo v and for k to be a whole number (clearly, v and s can not both be odd). Arya (1983) suggested a modified circulant plans for obtaining PDC. The analysis of PDC data based on circulant plans can be worked out through the software package SPAR1 (1991) developed by IASRI, New Delhi.
In second one, the association scheme of a partially balanced incomplete block (PBIB) design is used in constructing the PDC. PBIB designs are an important class of block designs that can be used in plant and animal breeding trials to investigate the genetic properties and potentials of inbred lines or individuals.

In the third approach for sampling the diallel cross, the average variance over all the comparisons is minimized as shown in Mathur and Narain (1976), Venkatesan (1985) and Singh and Hinkelmann (1990).

Furthermore, without going for mating design separately, one can go for a combined mating-environment design and the sample is obtained by making all possible crosses within blocks of a proper environmental design (PBIB design).

In this note, we prefer to adopt second method due to its simplicity, availability of a good number of PBIB designs that provides better choice, and relatively good efficiency of the selected plan. Gilbert (1958) pointed out the analogy between PDCs and incomplete block designs with blocks of size two. Since a complete diallel cross corresponds to a balanced incomplete block design (Kempthorne and Curnow, 1961), it is expected that partially balanced incomplete block designs will be related to certain types of PDCs. Kempthorne and Curnow (1961), Curnow (1963), Fyfe and Gilbert (1963) and Hinkelmann and Kempthorne (1963), Ghosh and Divecha (1997) gave some PDCs derived from PBIB with two associate classes.

The total number of crosses involved in a PDC using a two-associate-class association scheme is likely to be large resulting in difficulty to handle all of them effectively. Fyfe and Gilbert (1963) further introduced PDCs derived from a PBIB design with three-associate classes. If for given \( v \) (number of lines) there is a design with higher associate-classes, the number of crosses is likely to be small and there is more flexibility in the choice of associate classes [Das and Sivaram, 1968]. Das and Sivaram (1968) obtained plans for partial diallel crosses using PBIB designs with any block size, any values of \( \lambda \) and any number of associate classes. Some work on this aspect has been done by Arya and Narain (1977), Narain et al. (1974), Narain and Arya (1981), Agrawal (1985), Kaushik and Puri (1989), Narain (1993) and Kaushik (1999).

2. Efficient Mating Design
The association scheme of a \( m \)-associate-class PBIB design can be used to obtain mating designs by crossing each line with its \( m^{th} \) associates \((m=1,2,3,\ldots)\).

The different plans obtained are as follows:
Plan \( D_1 \): crosses of \( v \) lines with their first associates resulting in \( n_1v/2 \) crosses,
Plan \( D_2 \): crosses of \( v \) lines with their second associates resulting in \( n_2v/2 \) crosses, and
Plan \( D_m \): crosses of \( v \) lines with their third associates resulting in \( n_mv/2 \) crosses.

Among these plans, the one with maximum information per cross is selected. The following fixed effects model involving the gca effects of lines has been considered:

\[
Y_{ii'} = \mu + g_i + g_{i'} + e_{ii'}, \quad i < i' = 1, 2, \ldots, v
\]
where \( Y_{ii'} \) is the response obtained from the cross \((i \times i')\), \( \mu \) is the general mean and \( g_i \) (\( g_i' \)) is the gca effect of line \( i \) (\( i' \)), and \( e_{ii'} \) are errors, assumed to be identically and independently distributed with mean zero and constant variance \( \sigma^2 \). In matrix notation, the model can be written as,

\[
Y_m = \mu \mathbf{1} + X_m \mathbf{g} + \mathbf{e} \quad \ldots(2.2)
\]

where \( Y_m \) is the column vector of \( n_m v/2 \) observations \((m=1,2,3,...)\) depending on the partial diallel plan considered, \( \mathbf{1} \) is a \((n_m v/2) \times 1\) vector of unity, \( \mathbf{g} = (g_1, g_2, ..., g_v) \), \( X_m = ((x_{\alpha i})) \) is the \((n_m v/2) \times v\) design matrix with \( x_{\alpha i} = 1 \), if \( i^{th} \) line occurs in the \( \alpha^{th} \) cross \((\alpha = 1, ..., n_m v/2)\) and \( x_{\alpha i} = 0 \), otherwise and \( \mathbf{e} \) is a \((n_m v/2) \times 1\) vector of errors. The reduced normal equations for estimating gca effects from the \( m \) plans are seen to be:

\[
C_m \hat{\mathbf{g}} = Q_m \quad \ldots(2.3)
\]

where \( C_m = X' [\mathbf{I} - \frac{2}{n_m v} \mathbf{J}] X \), \( Q_m = X' (\mathbf{Y} - \bar{\mathbf{Y}}) \), \( m = 1, 2, 3, ... \)

with \( \mathbf{J} \) representing a matrix of unity elements of order \((n_m v)/2 \times (n_m v)/2\) and \( \mathbf{I} \) is an identity matrix of order \((n_m v)/2\). \( \bar{\mathbf{Y}} \) denotes the mean of the observations pertaining to the crosses considered. \( C_m \) is called the information matrix for estimating the gca effects and is singular with row and column sums equal to zero and rank \((C_m) \leq v\).

There will be \( m \) variances for testing the estimated differences \((\mathbf{g}_i - \mathbf{g}_i')\) according as the parents \( i \) and \( i' \) are first, second, or... \( m^{th} \) associates, from each \( D_m \), \( m = 1, 2, 3, ... \). The average variance of the difference between gca effects for the \( m^{th} \) plan is

\[
\frac{1}{V_{D_m}} = \frac{v n_1 V_{m1} + v n_2 V_{m2} + \ldots + v n_m V_{mm}}{v(v-1)/2} = \frac{n_1 V_{m1} + n_2 V_{m2} + \ldots + n_m V_{mm}}{n_1 + n_2 + \ldots + n_m} \quad \ldots(2.4)
\]

where \( V_{mm} \) is the variance of the difference between gca effects of two lines that are crossed, \( V_{mm'} \) \((m \neq m' = 1,2,3,...)\) is the variance between gca of two lines that are not crossed. These variances are obtained from the inverse of the information matrix as given in (2.3). The information obtained per cross from the \( m^{th} \) plan is

\[
\text{Inf}_m = \frac{2}{v n_m V_{D_m}} \quad \ldots(2.5)
\]

The efficiency factor in terms of the information per cross as compared to a complete diallel cross plan, assuming the error variance to be same for both the plans, is seen to be

\[
E_m = \frac{n_m V_{D_m}}{\sum_m n_m V} \quad \ldots(2.6)
\]
where $\bar{V}$ is the average variance of the estimate of the elementary contrast pertaining to gca effects in case of a complete diallel cross plan.

### 3. Analysis of Efficient Mating Design in Environmental Design

Experiments for PDC plans may be carried out in randomized complete block design, considering each cross as a treatment. However, if the number of crosses is large, accommodating all the crosses in a single block may result in large intra-block variance. Incomplete block designs may be used to reduce the block size (Singh and Hinkelmann 1995; Agarwal and Das 1990; and Sharma 1998). The method of accommodating the crosses from a PDC in balanced incomplete block design, as an environmental design requires, in general, a large number of replications. Alternatively, a 2-associate-class PBIB design (Singh and Hinkelmann 1995) with smaller number of blocks may be used, if such a design exists. Three-associate class PBIB design is another alternative that helps in reducing the size of the experiment.

The analysis of efficient partial diallel cross plans in complete and / or incomplete block designs can be worked out on the lines similar to Singh and Hinkelmann (1995), as explained below.

Consider a m-associate-class PBIB design with parameters $v^* = \frac{v_{me}}{2}$, $b^*$, $r^*$, $k^*$, $\lambda_1^*$, $\ldots$, $\lambda_m^*$, where $n_{me}$ is the $n_m$ (m=1,2,3,...) corresponding to most efficient PDC plan. An appropriate model for data from a PDC in PBIB design is

$$Y_{ijl} = \mu + g_i + g_j + \beta_l + e_{ijl} \quad \ldots(3.1)$$

where $Y_{ijl}$ refers to the observation from the $i \times j$ cross in block $l$, $\mu$ is the general mean, $g_i$ is the gca of line $i$ ($i = 1,2,\ldots,v$), $\beta_l$ is the effect of block $l$ ($l = 1,2,\ldots,b^*$), and $e_{ijl}$ is an error term following $N(0, \sigma^2 I)$. In matrix notation, the model becomes

$$Y = \mu 1 + X_g g + X_\beta \beta + e \quad \ldots(3.2)$$

where $Y$ is the $n \times 1$ vector of observations ($n = n_{me} b^*$), $1$ is an $n \times 1$ vector of unity elements, $X_g = (x_{gui})$ is an $n \times v$ matrix with elements $x_{gui} = 1$ if one of the parents of the cross on unit $u$ is line $i$, and 0, otherwise; $X_\beta = (x_{pul})$ is an $n \times b^*$ matrix with elements $x_{pul} = 1$ if unit $u$ is contained in block $l$, and 0, otherwise; $g = (g_1,g_2,\ldots,g_v)'$ is the vector of general combining abilities; $\beta = (\beta_1, \beta_2,\ldots, \beta_{b^*})$ is the vector of block effects; and $e$ is the $n \times 1$ vector of errors.

Let $\psi$ be the $n_{me} \times b^*$ cross vs. block incidence matrix and $Z$ is the cross vs. line $n_{me} \times v$ incidence matrix. The reduced normal equations for the gca effects are

$$C_{\gamma} \hat{g} = Q_g \quad \ldots(3.3)$$
where \( C_g = r^*Z'Z - \frac{1}{k^*}Z'\psi\psi'Z \), and

\[
Q_g = X_g'Y - \frac{1}{k^*}Z'\psi X'_{\beta}Y.
\] ...\( (3.4)\)

\[
\hat{g} = C_g^{-*}Q_g
\] ...\( (3.5)\)

Equation (3.9) can be solved to give

\[
\text{where } C_g^{-*} \text{ is a g-inverse of } C_g. \text{ The analysis of variance is as given below:}
\]

\[
\begin{array}{|c|c|c|}
\hline
\text{Source} & \text{d.f.} & \text{Sum of Squares} \\
\hline
\text{Blocks} & b^*-1 & (1/k^*)B'B - G^2/n \\
\hline
\text{gca} & v-1 & \hat{g}'Q_g \\
\hline
\text{Error} & n - b^* - v + 1 & \text{By subtraction} \\
\hline
\text{Total} & n - 1 & Y'Y - G^2/n \\
\hline
\end{array}
\]

where \( B \) denotes the vector of block totals and \( G \) denotes the grand total of all observations.

Now,

\[
c'\hat{g} = c'C_g^{-*}Q_g
\] ...\( (3.6)\)

is the least squares estimator for any contrast say, \( c'g \) among the gca effects with

\[
\text{Var}(c'\hat{g}) = c'C_g^{-*}c\sigma^2.
\] ...\( (3.7)\)

These variances can be worked out by using the PROC IML procedure of SAS.

References and Suggested Reading
Gilbert, N. (1958). Diallel crosses in plant breeding.\( Heredity, 12, 477-492.\)


